**Review of**[3]
**Quantum Information Theory**
**by Mark M. Wilde**
**Cambridge University Press, 2013**
**672 pages, Hardcover, $79.00**

.

**Review by**
**Subhayan Roy Moulick** subhayan@acm.org
**Department of Physical Sciences**
**Indian Institute of Science Education and Research, Kolkata, India**

# 1 Introduction

Over the last decade, quantum information theory has become one of the frontier areas of study among mathematicians, computer scientists, physicists, and engineers. This is because of the profound implications in communication and computational tasks that quantum physics offers, and that challenge our current classical conventions. Quantum information theory regards quantum states as a new form of information, and seeks to understand what quantum information is good for – things that classical computers cannot do. Mark Wilde's book aims to foster the very same and discusses the role of quantum information for communication tasks.

While there are several books and concrete references that have been written in the area over the years, this book offers a unique approach and focuses on quantum information theory alone. It includes the post-millennium results, which most others do not convey. The book is divided into six sections and, through twenty-five chapters, it introduces the subject and builds up a solid foundation with rigorous (but approachable) mathematics.

# 2 Summary

The first section gives an introduction to (quantum) Shannon theory and mentions the essentials and the history and development of the subject. The second chapter on classical Shannon theory gives a necessary brief introduction to the same.

The second section is where quantum theory makes its real appearance. Chapter 3 discusses the noiseless quantum theory and talks about qubits, gate operators, and measurements. The uncertainty principle, composite systems and their evolution, along with the famous no-cloning theorem, and the star of quantum theory—entanglement—is covered here. The chapter finishes with an extension to qudits, or higher dimension systems, which is a rather straightforward extension to include. This is followed by the famous Schmidt decomposition and CHSH games, all are well presented here. However, quantum systems are not always perfect, to say nothing of noiseless. This is not a matter of philosophy, but a hard fact when dealing with quantum systems. Thus Chapter 4 gives the noisy quantum theory, along the same lines as the preceding chapter. Ensembles, POVM, the partial trace, classical-quantum states, and very important examples of noisy evolutions as channels are new, among other things. The key takeaway from this chapter is the ensemble viewpoint which the author demonstrates beautifully, and which is to be a key tool for further study and research. The last chapter of this section discusses quantum theory from a radically different viewpoint, through the purification theorem. Loosely, this has to do with the idea that the uncertainty in quantum systems is due to their being entangled with other systems to which we do not have access. It talks about the purified quantum theory, and isometric extensions of the evolution of a system, quantum instruments, and measurement. This is one of those chapters that covers topics most books do not cover in their full glory.

Section 3 talks about the unit quantum protocols. Chapter 6 defines what it means to be a unit non-local quantum resource. Following that there are introductions to the protocols for entanglement distribution, superdense coding, and finally the very interesting, quantum teleportation. The arguments given here about their optimality using resource inequalities are interesting. In addition, again there is a straightforward extension to higher dimensions which is appended to the end of the chapter. Chapter 7 discusses coherent protocols through the implementation of the coherent bit channel, coherent dense coding, and coherent teleportation. A fundamental result, derived in this chapter, is the coherent communication identity. The protocols discussed in Chapter 6 are revisited in Chapter 8, to obtain the unit resource capacity region, and show that the unit resource achievable region consists of all linear combinations of the three protocols. Furthermore, it is shown that that is the best that one can do with the unit resources. From this, the direct and converse theorems are also derived.

Section 4 is where things get very interesting. It is devoted to introducing the mathematical toolkit required to study quantum Shannon theory. Chapter 9 is about distance measures, which allow comparison of quantum states and quantum channels. It discusses trace distance and fidelity from the very basics to the operational interpretations. It also talks about the relationship between the two. Gentle measurements, fidelity of noisy quantum channels and the Hilbert-Schmidt distance measure conclude the chapter. Information theory, right from the very first principles—information as a surprisal of an event to the ideas of entropy and information inequalities—are beautifully given in Chapter 10. This is perhaps my favorite chapter in the book, due to its condensed presentation without any compromise of rigor. Chapter 11 extends the ideas of classical information developed in the previous chapter to the quantum domain, along with discussions on the non-classical ideas of entropy being negative in conditional quantum entropy. This is quite profound in a sense, that we see entropy as the expected value of information content, which is always positive in the classical regime. The operational interpretation of conditional quantum entropy is interesting. However while not covered in the first edition, it is anticipated in the second edition of the book. I was quite impressed to read about continuity of quantum entropy, and the subsection on the uncertainty principle in the presence of quantum memories. While the preceding two chapters considered static entropic quantities, Chapter 12 is about the six dynamic entropic quantities for classical and quantum channels. These include mutual information of a classical and quantum channel, private information of a classical wiretap channel and quantum channel, Holevo information and coherent information of a quantum channel. The chapters that follow in this section are about classical typicality and are centered around the asymptotic equipartition property. These chapters build towards an asymptotic theory of information through elaborate discussions on weak and strong (joint and conditional) typicality. The application of typical sequences in compression and conditional typicality to Shannon's channel capacity theorem is also well explained here. In the same spirit, typicality in the asymptotic quantum domain, in the i.i.d. setting, is described in Chapter 14, in addition to definitions of quantum information source and typical subspace measurement. The last two chapters of this section cover the packing lemma and cover lemma. These two are in a sense opposite to each other. These are rather important and new results which have been developed recently, which is again quite unique to the book.

Section 5 consists of two chapters which study compression of information and entanglement concentration. Chapter 17 studies the quantum data compression theorem and the achievable rates, and proves both the direct coding theorem and the converse theorem for quantum data compression. It ends with an example and a brief discussion of other variations. The next chapter discusses how one can manipulate the "quantity" of entanglement using LOCC, through entanglement concentration. It starts out with a very clear example, and then formally defines entanglement concentration and proves it. The sections on common randomness compression and comparison with Schumacher compression are well worth the time.

The final section of the book deals with noisy quantum theory. This is where one encounters the gems of quantum Shannon theory. The Holevo-Schumacher-Westmoreland (HSW) theorem characterizes the classical capacity of a certain class of channels and is seen in Chapter 19. The classical capacity of the Hadamard channel, depolarizing channel, and superadditivity of Holevo information are also studied here. The next chapter in this section studies entanglement-assisted classical communication and asks how shared entanglement could be useful in transmitting classical information over a noisy channel. Following a detailed example, the chapter introduces the entanglement-assisted classical capacity theorem due to Bennett-Shor-Smolin-Thapliyal. I enjoyed the discussion on how (quantum) feedback does not increase (entangled assisted) classical capacity. It finally ends with a rather nice discussion of how to compute the entanglement-assisted classical communication with limited entanglement for the quantum erasure and amplitude damping channel. Chapter 20 discusses coherent communication with noisy resources. Chapter 22 and 23 consider the nature of information transmission over private channels, proves the private classical capacity theorem due to Devetak-Cai-Winter-Yeung, and the quantum capacity theorem, along with good examples. Discussions on quantum capacities and entanglement distillation conclude the chapter. Finally, the channel coding theorems discussed throughout the book are unified in Chapter 24, with the quantum dynamic capacity theorem, before concluding the book with yet another chapter containing the summary and outlook.

## 3 Opinion

My overall impression of the book is quite favorable. While at times I felt some parts were a bit tedious, I would still say the book does a phenomenal job of introducing, developing and nurturing a mathematical sense of quantum information processing. I also quite liked the coherent picture it paints. The steady presentation of the material from the ground up is done very well. While it may be useful for the reader already to have taken a course, or at least have had some exposure to quantum physics or information theory to fully appreciate the material, I am confident a motivated reader can do without. In a nutshell, this is an essential reference for students and researchers who work in the area or are trying to understand what it is that quantum information theorists study. Wilde, as mentioned in his book, beautifully illustrates "the ultimate capability of noisy physical systems, governed by the laws of quantum mechanics, to preserve information and correlations" through this book. I would strongly recommend it to anyone who plans to continue working in the field of quantum information.